Automatic discounting rate computation using a dissent criterion

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***Abstract*—An iterative method for automatic discounting rate computation is introduced in this paper. As part of a conjunctive combination of sources, discounting reduces the impact of a source on the result of the fusion. The key problems related to discounting are: what sources must be discounted and up to what degree. We propose a method that jointly tackles these two issues by computing rates using a dissent measure. The dissension of the sources is evaluated by comparing them to the average of their mass functions. Sources very different from others (like sources resulting from outliers) are thus heavily discounted. This discounting process is iterated so as to obtain a minimum distance between the conjunctive combination of the discounted sources and a categorical mass function. The efficiency of the method is evaluated through several simulations.**

Keywords: belief functions, discounting rates.

I. Introduction

The Dempster-Shafer theory [1], also known as evidence theory, is a mathematical framework that allows uncertain and imprecise data to be modelled and processed. This theory has gained popularity because it has proved to be efficient on various data fusion problems. Each source of information involved in the data fusion problem at stake collects pieces of evidence, yielding a mass function. These functions are then aggregated using a combination rule.

Many authors have thus worked on combination rules in order to provide a fusion tool that would produce results matching expectations. As expectations are rather subjective or application driven, designing a universal rule is virtually infeasible. Consequently, authors introducing a new rule usually proceed as follows: first identify what properties are required to resolve the fusion problem, and then design a rule that possesses such properties.

The most widely used rules are Dempster's rule and the conjunctive rule whose core principles are identical. They both aggregate sources conjunctively, *i.e.* belief mass redistributions is allowed if all the sources share common evidences. This property is very interesting because the output mass function is more informative than the initial ones. The conjunctive rule is the unnormalized version of Dempster's rule and was introduced by Smets in the Transferable Belief Model (TBM) [2]. In this article, we focus on the conjunctivity property and thus on these two rules.

Although these two rules are well founded theoretically, they have one major limitation: a lack of robustness with respect to conflicting evidences. This is why all sources are supposed to be reliable when performing a conjunctive combination. In many applications, it is difficult to design a system free of outliers. An outlier is likely to produce conflicting evidence, thereby violating the reliability hypothesis.

When sources are conflicting, a conjunctive combination assigns a large mass to the empty set $\emptyset$. This mass is also known as the degree of conflict. In Dempster's rule, the degree of belief is redistributed identically to all other masses. In the conjunctive rule, this degree is considered as a meaningful mass and is kept without any redistribution.

Several strategies exist to deal with an excessively large degree of conflict. First, the most simple is to consider that no reliable conclusion can be drawn from the collected data and thus no decision can be made. This solution is cautious but may not be acceptable for some systems in which the decision is of critical interest.

Another possibility is to redistribute the degree of conflict in a way that may impact less heavily on the output mass function. Yager's rule [3] transfers it directly to the set representing ignorance. Inagaki [4] and Lefevre *et al.* [5] designed a family of rules dealing with conflict reallocation. The main idea behind these approaches is to identify which member of the rule family is optimal regarding an appropriate criterion. Using non-linear functions, Dezert and Smarandache's PCR5 [6] redistributes conflict to the subsets from which it was generated. These solutions have proved to be efficient in particular contexts but they imply a modification of the conjunctive or Dempster's rule inner process.

Indeed, it may also be argued that applying conjunctive rules to conflicting sources is simply irrelevant and that the mass function models[1] should then be revised [7]. This implies notably that outliers can be dodged at an early stage of the mass function construction. However the amount of collected data may not always be sufficient to design of a fully robust mass function model.

An intermediate approach consists in revising the mass function themselves instead of revising the models. In this article we follow this strategy. As part of the Dempster-Shafer theory, discounting is a tool that revises the mass function of a source by lowering the certainty of the information carried by the

[1]a mathematical process that derives mass functions from raw data.

source. In Ref. [8], Smets introduces an expert system that comprises several combination and discounting steps in order to respond to situations with large conflicts.
When one decides to implement a discounting step, a two-fold problem must be dealt with:

- What sources must be discounted? (identification)
- Up to what degree should the identified sources be discounted? (valuation)

We propose to tackle directly the valuation issue and to consider that the identification step can be skipped provided that the computed discounting coefficient for reliable sources is negligible as compared to those of unreliable sources. We thus introduce an iterative method that automatically evaluates optimal coefficient regarding a dissent criterion.
Our approach is novel in the sense that no meta-information on the reliability of the source is needed. The only assumption on which our method relies is the following: the more a source is in conflict with the majority opinion, the more heavily it must be discounted.
In the next section, we remind of some classical Dempster-Shafer theory definitions. Following that, our new iterative method for discounting coefficient calculation is presented and justified. In the final section, the efficiency of our approach is demonstrated on several bba sets with various degrees of certainty and proportions of conflicting sources.

II. Dempster-Shafer Theory

A. Fundamental concepts

Dempster-Shafer Theory (DST) provides a formal framework for dealing with both imprecise and uncertain data. The finite set of mutually exclusive solutions is denoted by $\Omega = \{\omega_1, ..., \omega_K\}$ and is called the frame of discernment. The set of all subsets of Ω is denoted by 2^Ω. A source S collects pieces of evidence leading to the assignment of belief masses to some elements of 2^Ω. The mass of belief assigned to A by S is denoted $m\left[S\right]\left(A\right)$. For the sake of simplicity, the notation $m\left[S_1\right]$ is replaced by m_1 hereafter. The function $m : 2^\Omega \rightarrow [0, 1]$ is called **basic belief assignment** (**bba**) and is such that $\sum_{A\subseteq\Omega} m\left(A\right) = 1$. A set A such that $m\left(A\right) > 0$ is called a **focal element**. Two elements of 2^Ω represents hypotheses with noteworthy interpretations:

- $\emptyset$: the solution of the problem may not lie within Ω.
- Ω: the problem's solution lies in Ω but is undetermined.

The open-world assumption states that $m\left(\emptyset\right) > 0$ is possible. The closed-world assumption bans $\emptyset$ from any belief assignments. Under the closed-world assumption, the standard way of combining distinct[2] pieces of evidence m_1 and m_2 is Dempster's combination rule $\oplus$: $\forall A \neq \emptyset$,

$$m_\oplus\left(A\right) = \frac{1}{1-\kappa} \sum_{\substack{B,C\subseteq\Omega \\ B\cap C=A}} m_1\left(B\right) m_2\left(C\right), \quad (1)$$

$$\text{with } \kappa = \sum_{\substack{B,C\subseteq\Omega \\ B\cap C=\emptyset}} m_1\left(B\right) m_2\left(C\right). \quad (2)$$

The mass κ is called the degree of conflict. The open-world counterpart of Dempster's rule is the conjunctive rule $\cap$: $\forall A \subseteq \Omega$,

$$m_{\cap}\left(A\right) = \sum_{\substack{B,C\subseteq\Omega \\ B\cap C=A}} m_1\left(B\right) m_2\left(C\right). \quad (3)$$

Along with mass function combination, it is also possible to revise beliefs thanks to a process named discounting [1]. Discounting with discount rate $\alpha \in [0, 1]$ is defined as:

$$m^\alpha\left(X\right) = \begin{cases} (1-\alpha)m\left(X\right) \text{ if } X \neq \Omega, \\ (1-\alpha)m\left(X\right) + \alpha \text{ if } X = \Omega. \end{cases} \quad (4)$$

The higher α is, the stronger the discounting. Thanks to discounting, an unreliable source's bba is transformed into a function assigning a larger mass to Ω.

B. Related works

A classical data fusion processing using DST consists of, first, obtaining bbas thanks a bba model and raw data, then, discounting some of the bbas according to bba or data analysis and finally aggregate the bbas using an appropriate combination rule. As exposed in the introduction, the discounting step can be applied if sources to be discounted are identified and if their discounting rates have been evaluated.
Most of the time, discounting rates are hand-tuned but some authors have proposed several methods to obtain them without supervision. In [9], Smets computes discounting rates using a error function minimization. This method is dedicated to data classification and requires a set labelled data. In [10], Martin *et al.* introduce a discounting rate evaluation method that relies only on bba values themselves. Their approach is based on a measure of conflict between two bbas defined by:

$$Conf\left(m_1, m_2\right) = d_{BPA}\left(m_1, m_2\right) \quad (5)$$

with d_{BPA} a bba distance introduced by Jousselme *et al.* in [11] and defined as: $d_{BPA}\left(m_1, m_2\right) = \sqrt{1/2\left(\vec{m_1} - \vec{m_2}\right)^t D\left(\vec{m_1} - \vec{m_2}\right)}$ with $\vec{m}$ a vector form of the bba m and D a $2^N \times 2^N$ matrix whose elements are $D\left(A, B\right) = \left|A \cap B\right| / \left|A \cup B\right|$. When dealing with a larger set of bbas $S = \{m_1, ..., m_M\}$, the authors investigate several ways to compute the measure $Conf$:

$$Conf\left(m_i, S\right) = \frac{1}{M-1} \sum_{j=1, i\neq j}^{M} d_{BPA}\left(m_i, m_j\right) \quad (6)$$

$$\text{or } Conf\left(m_i, S\right) = d_{BPA}\left(m_i, m_*\right) \quad (7)$$

[2]Pieces of evidence are distinct if the construction of beliefs according to one piece of evidence does not restrict the construction of beliefs using another piece of evidence.

with m_* the combination of all bbas of S except m_i. m_* can be obtained using different combination rules. Once the conflict measure chosen, the authors further propose to compute discounting rates as follows:

$$\alpha_i \quad = \quad f\left(Conf\left(m_i, S\right)\right) \tag{8}$$

with α_i the discounting rate to apply to m_i and f a decreasing function.

In this article, we have developed independently an iterative method that is based on a similar procedure as the one of Martin *et al*. We first compute coefficients $\left\{\alpha_i^0\right\}_{i=1}^M$ for each member of the initial set S using equation (8) with the identity function as function f but then, in contrast to Martin *et al*., we iterate the process on the discounted set of bbas $S^1 = \left\{m_1^{\alpha_1^0}, ..., m_M^{\alpha_M^0}\right\}$ and thus obtain new coefficients $\left\{\alpha_i^1\right\}_{i=1}^M$. Under a few conditions on the discounting rates calculation, the method converges as the rate values increase more and more slowly.

To identify the optimal set of discounting rates $\left\{\alpha_1^{i_{opt}}, ..., \alpha_M^{i_{opt}}\right\}$ among those computed at each iteration, an *a posteriori* analysis step is employed. We examine the conjunctive combinations obtained at each step and compare them to categorical bbas[3] using the distance d_{BPA}. The iteration yielding the minimum distance is i_{opt}. Figure 1 summarizes our approach.

Our approach prevents from choosing function f and tuning its parameters. A comparison of the performances of our approach and Martin *et al*.'s approach can be found in section IV.

Note that Schubert [12] also introduced an iterative approach for discounting rates computation. At step j the rate of source S_i is computed as follows : $\alpha_i^j = \frac{1-m_{\textcircled{\cap}}^{j-1}(\emptyset)}{1-m_{i*}^{j-1}(\emptyset)}$ with $m_{\textcircled{\cap}}^j$ the conjunctive combination of $\left\{m_1^{\alpha_1^j}, ..., m_M^{\alpha_M^j}\right\}$ and m_{i*}^j the result of the same combination without $m_i^{\alpha_i^j}$. This formula is less robust than a distance-based one because when there is more than one source in conflict with the majority opinion, removing a conflicting source from the conjunctive combination will not yield a steep reduction of the degree of conflict. Furthermore, the method is iterated until the degree of conflict gets beneath a predefined threshold but there is no proof of convergence that would guarantee that any threshold can be reached.

[3]a bba assigning mass 1 to a singleton.

III. Iterative method for automatic discounting rate computation

A. Initial settings

As explained in the introduction, the advantage of our approach and Martin *et al*.'s approach is that the only assumption on which they rely is that "the majority opinion is the actual one". This said, defining what the majority opinion is as part of DST is not a trivial task. In [13], Murphy proposed to use the mean of bbas arguing that the properties of the mean are better suited for conflicting evidences:

$$m_{mean} = \frac{1}{M} \sum_{i=1}^{M} m_i \tag{9}$$

Indeed if a subset s_1 of S corresponds to a cluster of concordant bbas and if this subset contains more bbas than any other cluster, then m_{mean} is likely to be close to the bbas composing s_1. In conclusion, m_{mean} can be used as an estimation of the majority opinion. We thus propose to evaluate a first set of discounting rates as follows:

$$\alpha_i^0 = d_{BPA}\left(m_i, m_{mean}\right) \tag{10}$$

Equation (10) yields low rates for bbas close to the mean (those in accordance with the majority opinion) and high rates

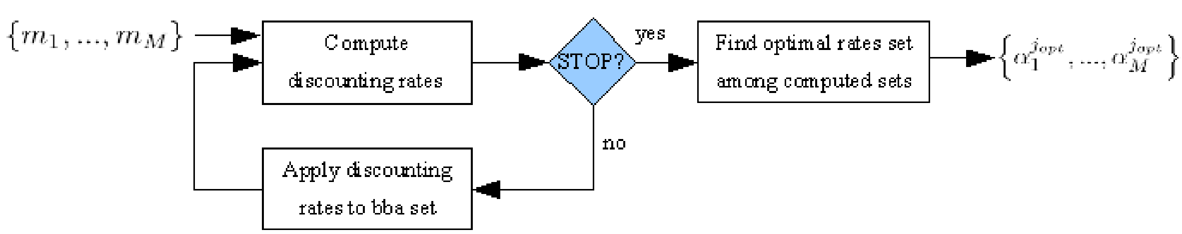

Figure 1. Iterative procedure for discounting rates computation.

for bbas afar from the mean (those which are the cause of the dissent).

Note that the mean of bbas is one of the combination techniques investigated in [10] but equation (10) is slightly different from equation (7) as we do not discard m_i from the calculation of m_{mean}. Removing it produces a significant difference only in the case of isolated bbas. In this article, it is preferred to allow a singular bba to impact on the results. The presence of an outlier-bba should (at least even slightly) alter the certainty of the bba obtained after combination.

B. Iterating the process

The relative values of the rates impacts on the fusion result at least as much as the absolute values of the rates. In other words, it is not sufficient to have high valued rates for unreliable sources, it is also necessary that the difference between rates of reliable and unreliable source be large enough. Consequently, the relevance of the values of the rates $\left\{\alpha_i^0\right\}_{i=1}^M$ can be questioned. Should the difference between unreliable and reliable bbas be stressed or softened? And by what mean can it be done?

Martin *et al.*'s answer to this issue is function f but they do not specify what is the optimal function or how to set the parameter of the function. To avoid such problems, we propose to discount the initial set of bbas and recalculate new rates on this new set of bbas using the method described in the previous subsection.

By iterating this process, successive sets of discounting rates are produced and can be analysed afterwards to determine which set is optimal according to a criterion. An iterative procedure implies successive discounting on the original bbas. m^{α^0,α^1} denotes bba m^{α^0} discounted with rate α^1. Note that successive discountings with rates $\left\{\alpha^0,...,\alpha^K\right\}$ can be summarized into a global one with rate β^K:

$$\beta^K = 1-\prod_{i=0}^{K}\left(1-\alpha^i\right) \quad (11)$$

$$\text{and } \beta^K = \beta^{K-1}\left(1-\alpha^K\right)+\alpha^K. \quad (12)$$

The main difficulty regarding the implementation of such an iterative method is to determine a stopping condition $\mathfrak{C}$. If the series α_i^j are decreasing w.r.t. j, it can easily be proved that the series $\Delta\beta_i^j = \beta_i^{j+1}-\beta_i^j$ are also decreasing w.r.t. j. Consequently, the following condition $\mathfrak{C}$ can be used to stop iterations:

$$\mathfrak{C} = OR_{i=1}^{M}\left(\beta_i^j > \epsilon\right) \quad (13)$$

with ϵ a classical stopping parameter which is related to the desired precision on the rates and OR the logical *or*. To ensure that all series α_i^j are decreasing, we use:

$$\forall j>0, \alpha_i^j = \alpha_i^{j-1} d_{BPA}\left(m_i^{\beta_i^{j-1}}, m_{mean}\right) \quad (14)$$

The discounted rates computation procedure is summarized by algorithm 1.

Algorithm 1 Iterative discounting rates computation (IDRC)

```
for i = 1 to M do
  α_i^0 ⟵ d_BPA(m_i, m_mean)
end for
j ⟵ 0
while condition ℭ is true do
  for i = 1 to M do
    Compute discounted bba m_i^{β_i^{j-1}} = m_i^{α_i^0,...,α_i^{j-1}}
    α_i^j ⟵ α_i^{j-1} d_BPA(m_i^{β_i^{j-1}}, m_mean)
  end for
  j ⟵ j + 1
end while
```

C. Analysis step

At this stage, several sets of discounting rates have been computed and the most relevant set must be determined. First, as explained in the introduction, it is intended here to process the bbas using the conjunctive rule so as to obtain a more committed[4] bba after combination. Consequently, the series of the conjunctive combinations $m_{\odot}^j = \odot_{i=1}^M m_i^{\beta^j}$ must be screened in order to identify the optimal iteration j_{opt}.

A plethora of commitment measures can be found in the literature [15]–[24], proving that it is not easy to jointly weigh imprecision and uncertainty as part of a single measure. If the initial bbas were not conflicting and sufficiently precise the output bba would be close to a categorical bba $\{\omega_i\}^0$, *i.e.* a bba assigning a mass 1 to only one focal element $\{\omega_i\}$. A categorical bba is the most committed kind of bba, therefore we propose to evaluate the commitment of a bba m by computing the minimal distance between m and categorical bbas. Consequently, we have:

$$j_{opt} = argmin_j\left[\min_{i,j} d_{BPA}\left(\{\omega_i\}^0, m_{\odot}^j\right)\right] \quad (15)$$

The iterative discounting rate computation is now complete. Its efficiency is tested on various sets of bbas in the next section.

IV. Results and discussions

In this section, our iterative discounting rate computation (IDRC) method is tested and compared to other techniques. The bbas used in the experiments are generated randomly on the basis of the bba model of Appriou [25].

A. Robustness to the proportion of unreliable sources

To validate our approach, it is important to monitor its behaviour regarding the proportion of unreliable sources among the total number of bbas to aggregate. In this experiment, sets of 100 bbas are aggregated: x of them are unreliable and $100-x$ are reliable. The reliable sources assign a large mass to a singleton denoted $\{a\}$. The unreliable sources assign a large mass to another singleton denoted $\{b\}$. Table I shows one of the reliable bbas (assigning a large mass to $\{a\}$) and one of the unreliable bbas (assigning a large mass to $\{b\}$).

Figure 2 shows the performances in terms of conflict reduction and discounting rates computation for both IDRC

Table I
BBA EXAMPLES (RELIABLE AND UNRELIABLE)

	$\emptyset$	$\{c\}$	$\{a\}$	$\{a,c\}$	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\{\Omega\}$
reliable bba	0	0.0010	0.9979	0	0.0010	0	0	0
unreliable bba	0	0.0011	0.0010	0	0.9977	0	0	0

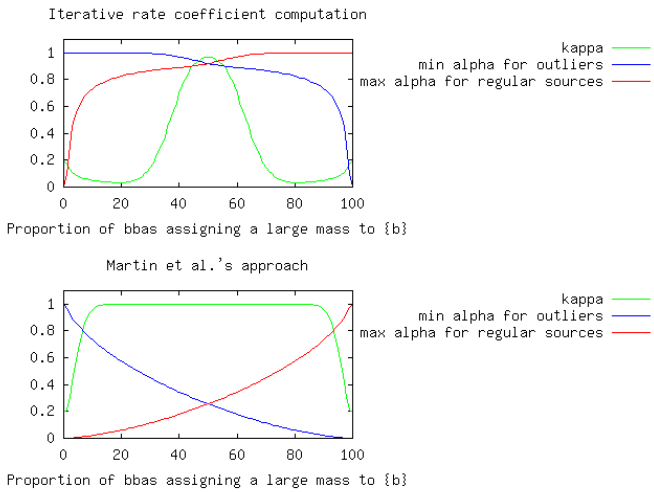

Figure 2. Comparison of IDRC and Matin *et al.*'s approach when x varies.

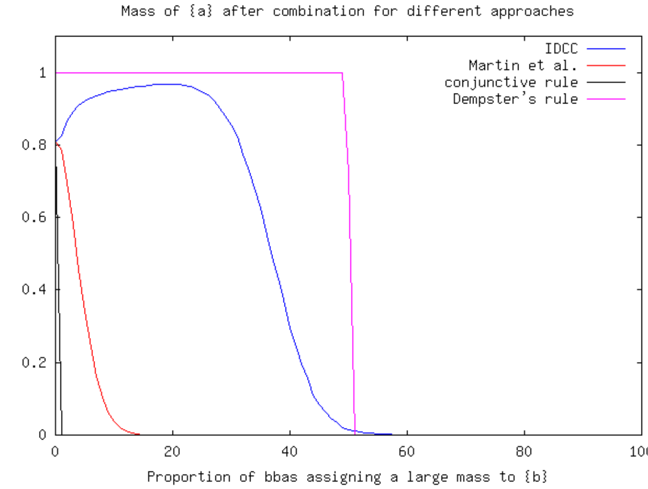

Figure 3. Comparison of IDRC, $\textcircled{\cap}$, $\oplus$ and Matin *et al.*'s approach when x varies.

Table II
BBA EXAMPLES WITH LOW AND HIGH NOISE

	$\emptyset$	$\{c\}$	$\{a\}$	$\{a,c\}$	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\{\Omega\}$
low noise bba	0	0.0021	0.9958	0	0.0020	0	0	0
high noise bba	0	0.0181	0.1436	0.1328	0.0180	0.0664	0.1328	0.4879

and Martin *et al.*'s approach. The proportion x varies between 0 and 100, consequently when x exceeds 50 the sources advocating for $\{b\}$ become the reliable ones in the sense that they carry the majority opinion. When $x = 50$, a borderline case is met. In such a situation, IDRC and Martin *et al.*'s approach both assign identical rates to all the sources and the degree of conflict remains very large which is the expected behaviour. However, as the proportion of unreliable sources goes over 10% Martin *et al.*'s approach is no longer able to reduce significantly the degree of belief.

On figure 3, the mass assigned to $\{a\}$ w.r.t. x is displayed for the conjunctive rule, Dempster's rule, Martin *et al.*'s approach and IDRC. Both Martin *et al.*'s approach and IDRC appears to be intermediate response to the fusion problem as compared to the conjunctive and Dempster's rules. The fact that $m_{IDRC}(\{a\})$ increases between $x = 0\%$ and $x = 20\%$ can be questioned. This is due to the fact that IDRC removes not only the conflict that is due to the dissent of the sources but also a part of the conflict that is due the non-idempotent aspect of conjunctive rules. In [10], this latter part of the conflict is called the *auto-conflict*. As the *auto-conflict* is not meaningful in terms of source unreliability, there is no information loss when removing it. The *auto-conflict* can evaluated when $x = 0$, it would be thus possible to post-process IDRC results in order to fully remove it and thus obtain a decreasing curve. Such a post-processing could be a simple conflict redistribution similar to the one of Dempster's rule.

[4]A definition of commitment of belief functions can be found in [14].

B. Robustness to noise

We now investigate the behaviour of the tested approaches when the proportion of unreliable sources is stable but the level of noise varies. In this experiment, the proportion of unreliable sources is set to 20% and the number of sources is 100. The noise variation results in bbas that assign significant weights to all singletons. Table II gives an example of a bba with low noise and an example of a bba with high noise. To display the results, we use a measure μ that is noise-dependent and defined as follows:

$$\mu = \sum_{i=1}^{80} m_i \left[reliable\right](\{a\}) - m_i \left[reliable\right](\{b\}) \qquad (16)$$

μ is the average difference between the masses allocated to $\{a\}$ and $\{b\}$ by reliable sources (for reliable sources $m(\{a\}) > m(\{b\})$). Figure 4 shows the performances in terms of conflict reduction and discounting rates computation for both IDRC and Martin *et al.*'s approach. Martin *et al.*'s approach does not reduce the degree of conflict whatever the level of noise is, because 20% of unreliable sources is already too much for it to cope with. IDRC reduces the degree of conflict when the level of noise is low. When the level is too high the conflict cannot be further reduced as the amount of *auto-conflict* is very large.

C. A broader case study

In this experiment, it is intended to aggregate sources with more disparity in their focal elements. The set of bbas to aggregate is made of 6 elements. Table III displays these bbas as well as the output bbas obtained using Dempster's rule, the

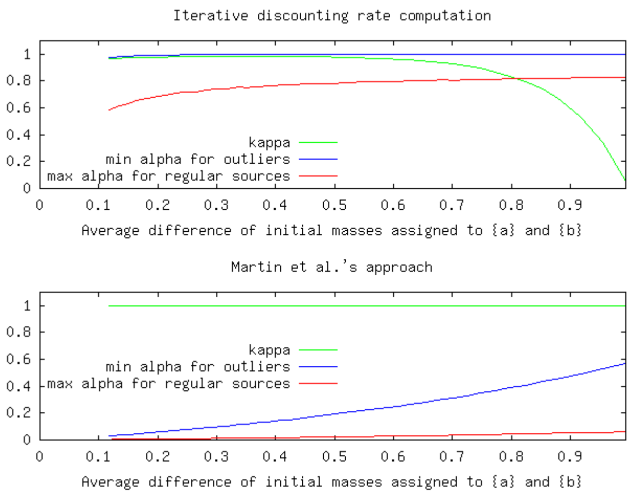

Figure 4. Comparison of IDRC and Matin *et al.*'s approach when noise varies.

Table III
BBA SET TO AGGREGATE AND THE OUTPUT BBAS FOR SEVERAL COMBINATION TECHNIQUES

	$\emptyset$	$\{c\}$	$\{a\}$	$\{a,c\}$	$\{b\}$	$\{b,c\}$	$\{a,b\}$	$\{\Omega\}$
m_1	0	0.0644	0.8341	0.0141	0.0644	0.0070	0.0141	0.0015
m_2	0	0.0647	0.8339	0.0141	0.0643	0.0070	0.0141	0.0015
m_3	0	0	0	0.9064	0.0767	0	0	0.0168
m_4	0	0.0772	0	0	0	0	0.9058	0.0168
m_5	0	0.0641	0.0646	0.0070	0.8342	0.0141	0.0141	0.0015
m_6	0	0.8332	0.0661	0.0141	0.0636	0.014	0.0070	0.0015
$m_{\oplus}$	0	0.0097	0.9803	0	0.0098	0	0	0
$m_{\cap}$	0.9950	0	0.0048	0	0	0	0	0
m_{Martin}	0.8599	0.0067	0.1239	0.0011	0.0066	0	0.0011	0.0003
m_{IDRC}	0.3163	0.0734	0.3089	0.0523	0.0735	0.0019	0.0523	0.1211

conjunctive, Martin *et al.*'s approach and IDRC. The results of Table III show that IDRC allows a steeper reduction of the degree of conflict than Martin *et al.*'s approach. Again, both of these approaches deliver intermediate solution as compared to Dempster's rule and the conjunctive rule.

V. Conclusion

In this article, a new iterative discounting rate computation (IDRC) technique is presented. It has the advantage to directly deliver discounting rates without needing first to identify what sources must be discounted. Furthermore, there is no additional information required to apply the method. IDRC relies on the assumption that the majority opinion is the actual solution, therefore bbas that far from this majority opinion are heavily discounted.

IDRC is compared to a reference method in the field of automatic discounting rate computation that was proposed by Martin *et al.* [10]. IDRC appears to produce results significantly different as compared to this method. IDRC allows notably to reduce the degree of conflict even if the proportion of unreliable sources is large. In future work, it is intended to investigate the possibility to further combine Martin *et al.*'s approach with ours. For example, it is possible to use a function f as well during the rate calculation in each step of IDRC. This may accelerate the convergence of IDRC without deteriorating discounting rate precision.

References

[1] G. Shafer, *A Mathematical Theory of Evidence*. Princeton Univ.press, 1976.
[2] P. Smets, "The combination of evidence in the transferable belief model," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 12, no. 5, pp. 447–458, 1990.
[3] R. Yager, "On the Dempster-Shafer framework and new combination rules," *Information Sciences*, vol. 41, pp. 93–138, 1987.
[4] T. Inagaki, "Interdependence between safety-control policy and multiple-sensor schemes via Dempster-Shafer theory," *IEEE trans. on Reliability*, vol. 40, no. 2, pp. 182–188, 1991.
[5] E. Lefevre, O. Colot, and P. Vannoorenberghe, "Belief function combination and conflict management," *Information Fusion*, vol. 3, pp. 149–162, 2002.
[6] F. Smarandache and J. Dezert, "Information fusion based on new proportional conflict redistribution rules," in *Int. Conf. on Information Fusion (FUSION'05)*, Philadelphia (USA), 25-29 july 2005, pp. 1–8.
[7] R. Haenni, "Shedding new light on Zadeh's criticism of Dempster's rule of combination," in *Int. Conf. on Information Fusion (FUSION'05)*, Philadelphia (USA), 2005, pp. 1–6.
[8] P. Smets, "Analyzing the combination of conflicting belief functions," *Information Fusion*, vol. 8, pp. 387–412, 2006.
[9] ——, "Data fusion in the transferable belief model," in *IEEE Int. Conf. on Information Fusion (FUSION'00)*, 2000, pp. 21–33.
[10] A. Martin, A.-L. Jouselme, and C. Osswald, "Conflict measure for the discounting operation on belief functions," in *IEEE Int. Conf. on Information Fusion (FUSION'08)*, 2008, pp. 1–8.
[11] A.-L. Jousselme, D. Grenier, and E. Bosse, "A new distance between two bodies of evidence," *Information Fusion*, vol. 2, no. 2, pp. 91–101, 2001.
[12] J. Schubert, "Conflict management in dempster-shafer theory by sequential discounting using the degree of falsity," in *Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU'08)*, 2008, pp. 298–305.
[13] C. Murphy, "Combining belief functions with evidence conflicts," *Decision Support Systems*, vol. 29, pp. 1–9, 2000.
[14] P. Smets and B. Ristic, "Kalman filter and joint tracking and classification based on belief functions in the TBM framework," *Information Fusion*, vol. 8, pp. 16–27, 2007.
[15] D. Dubois and H. Prade., "A note on measures of specificity for fuzzy sets," *International Journal of General Systems*, vol. 10, pp. 279–283, 1985.
[16] T. George and N. Pal, "Quantification of conflict in the Dempster-Shafer framework," *Int. Journal of General Systems*, vol. 24, no. 4, pp. 407–423, 1994.
[17] D. Harmanec and G. Klir, "Measuring total uncertainty in Dempster-Shafer theory: A novel approach," *Int. Journal of General Systems*, vol. 22, no. 4, pp. 405–419, 1994.
[18] U. Höhle, "Entropy with respect to plausibility measures," in *12th IEEE Int. Symposium on Multiple-Valued Logic*, 1982, pp. 167–169.
[19] G. Klir and J. Wierman, *Uncertainty-Based information: elements of generalized information theory*. New-York (USA): Physica-Verlag, 1998.
[20] Y. Maeda and H. Ichihashi, "An uncertainty measure with monoticity under the random set of inclusion," *Int. Journal of General Systems*, vol. 21, no. 4, pp. 379–392, 1993.
[21] M. T. Lamata and S. Moral, "Measures of entropy in the theory of evidence," *Int.Journal of General Systems*, vol. 14, no. 4, pp. 297–305, 1988.
[22] A. Ramer and G. Klir, "Measures of discord in the Dempster-Shafer theory," *Information Sciences*, vol. 67, no. 1 and 2, pp. 35–50, 1993.
[23] J. Vejnarova and G. Klir, "Measure of strife in Dempster-Shafer theory," *int. Journal of General Systems*, vol. 22, no. 1, pp. 25–42, 1993.
[24] R. Yager, "Entropy and specificity in a mathematical theory of evidence," *Int. Journal of General Systems*, vol. 9, pp. 249–260, 1983.
[25] A. Appriou, "Probabilités et incertitude en fusion de données multi-senseurs," *Revue scientifique de la defense*, vol. 11, pp. 27–40, 1991.